

CLASS : XI
SUBJECT: MATHEMATICS

M:M : 100
TIME: 3 HRS

General Instructions :

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Sections A comprises of 10 questions of one mark each, sections B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
4. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section-A

1. If $\mu = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{5, 6, 7\}$ and $B = \{5, 7, 8, 9\}$, then find $(A \cup B)'$.
2. Find the most general value of θ which satisfy equation $\sin \theta = \frac{-1}{2}$ and $\tan \theta = \frac{1}{\sqrt{3}}$.
3. Find the maximum value of $12 \sin \theta - 9 \sin^2 \theta$.
4. Find the equation of the ellipse the extremities of whose minor axis are (3,1) and (3,5) and whose eccentricity is $\frac{1}{2}$.
5. If $f(x) = \log \left(\frac{1+x}{1-x} \right)$ and $f \left(\frac{2x}{1+x^2} \right) = k f(x)$, the find k.
6. Solve the inequation: $|x-2| + |x+2| < 5$.
7. Sum the series $\frac{1}{3.7} + \frac{1}{7.11} + \frac{1}{11.15} + \dots$ to n term.
8. Find the probability of having 53 Sundays in a leap year.
9. Draw a rough sketch for $f(x) = |x-2| + |x+2|$.
10. Find $\frac{dy}{dx}$ if $y = 2^x \cdot \sin 3x$.

Section-B

11. In a survey of 100 students, the number of students studying the various languages were found as English only 18, English but not Hindi 23, English and Panjabi 8, English 26, Panjabi 48, Panjabi and Hindi 8 and No language 24. Find (i) How many students were studying Hindi? (ii) How many students were studying English and Hindi? (iii) Only one of the subjects?
12. If $A = \{1, 2, 3\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$ verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
13. Find the domain of the function.
 $\sqrt{f(x) = \log_7 \left[\log_9 \left(\log_3 \left(18x - x^2 - 77 \right) \right) \right]}$
14. In a ΔABC , prove that $a^3 \cos(B-C) + b^3 \cos(C-A) + c^3 \cos(A-B) = 3abc$.
OR
In an triangle ABC, if $a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{2}$, show that $\cot \frac{A}{2}, \cot \frac{B}{2}, \cot \frac{C}{2}$ are in A.P.
15. If $A + B + C = 180^\circ$, prove that $\sin 2A + \sin 2C - \sin 2B = 4 \cos A \cos B \sin C$.
16. Solve $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$.

17. If the lines $y = 3x + 1$ and $2y = x + 3$ are equally inclined to the line $y = mx + 4$, find the value of m .

OR

The vertices of a ΔABC are $(4,3,1)$, $B(6,4,3)$ and $C(-8,7,-2)$. The internal bisector of angle A meets the side BC in D. Find the coordinates of D.

18. Evaluate $\lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$, if α, β the roots of $ax^2 + bx + c = 0$.

OR

Evaluate $\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{2\sqrt{2} - (\cos \theta + \sin \theta)^3}{1 - \sin 2\theta}$

19. In an examination a candidate has to score a minimum qualifying marks in all the 5 subjects to pass. In how many ways can a student fail.

20. Find the sum to n terms of the series $1 + (1+2) + (1+2+3) + \dots$ to n terms.

21. Differentiate $\sqrt{\sin x}$ from first principle.

22. Two cards are drawn from a pack of 52 well shuffled playing cards at random. Find the probability that both the cards are red or queens.

OR

If cards are dealt one by one and kept aside until the third queen appears. Find the probability that the drawing process ceases in the 20th draw.

Section-C

23. Prove that $\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{14\pi}{15} = \frac{1}{16}$

24. By using principle of mathematical induction prove that

$$1.3 + 3.5 + 5.7 + \dots + (2n-1)(2n+1) = \frac{n(n^2 + 6n - 1)}{3}$$

25. Show that $3x^2 + 4y^2 - 12x - 8y + 4 = 0$ represents an ellipse. Find its centre, lengths and equations of axes, eccentricity, foci and directrices.

OR

For what value of C are the points $(2,0)$, $(0,1)$, $(4,5)$ and $(0,C)$ concyclic?

26. Calculate mean deviation (from median) for the following data:

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	8	15	16	6

28. (a) Three numbers in G.P. add up to 70. If the extremes be each multiplied by 4 and the mean by 5 they form an A.P. Find the numbers. (3)

(b) If a is the A.M. between b and c and G_1 and G_2 the two Geometric means between b and c then Prove that $G_1^3 + G_2^3 = 2abc$ (3)

OR

If $1 + 4x + 7x^2 + 10x^3 + \dots \infty = \frac{35}{16}$, then find x .

29. (a) If $x = -5 + 2\sqrt{-4}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4$. (3)

(b) Find the square root of $-7 - 24i$ (3)